# A SIMPLE APPROACH FOR CALCULATING THE POTENTIAL YIELD OF KRILL FROM BIOMASS SURVEY RESULTS 

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#### Abstract

Estimates are calculated for the parameter $\lambda$ in the yield equation $\mathbf{Y}=\lambda \mathrm{MB}_{0}$ for a krill fishery in which both the fishery and krill growth are seasonal. The criterion used for the calculations is that the probability that the krill spawning biomass drops below $20 \%$ of its median pre-exploitation level over a 20 year period of harvesting should equal $10 \%$. The value of $\lambda$ depends strongly on the value of $\mathbf{M}$ (natural mortality) and $\sigma_{\mathrm{R}}$ (recruitment variability), and also on the relative values of the ages at maturity and first-capture. Seasonal effects seem to be of lesser importance.


## Résumé

Calculs d'estimations du paramètre $\lambda$ dans l'équation de rendement $\mathbf{Y}=\lambda \mathrm{MB}_{0}$ pour une pêcherie de krill dans laquelle l'intensité de la pêche et la croissance du krill sont saisonnières. Pour les calculs, le critère utilisé est le suivant : la probabilité qu'en une période d'exploitation de 20 ans, la biomasse reproductrice du krill chute en dessous de $20 \%$ de son niveau moyen d'avant l'exploitation, devrait être égale à $10 \%$. La valeur de $\lambda$ est fortement dépendante de la valeur de $\mathbf{M}$ (mortalité naturelle) et de $\sigma_{\mathrm{R}}$ (variabilité du recrutement) ainsi que des valeurs relatives des âges de maturité et de première capture. Les effets saisonniers semblent être de moindre importance.

## Резюме

Вычислены оценки параметра $\lambda$ в уравнении вылова $\mathbf{Y}=$ $\lambda \mathrm{MB}_{0}$ в промысле криля, при котором как промысел, так и рост криля носят сезонный характер. Критерием вычислений являлось то, что вероятность спада биомассы нерестующего криля ниже $20 \%$ его медианного доэксплуатационного уровня за 20-летний период промысла, должна равняться $10 \%$. Значение $\lambda$ существенно зависит от значения $M$ (естественная смертность) и $\sigma_{R}$ (изменчивость пополнения), а также от относительных значений возраста при половозрелости и первом вылове. Сезонные воздействия, по-видимому, представляют меньшую важность.

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## Resumen

Se calculan las estimaciones del parámetro $\lambda$ en la ecuación de rendimiento $\mathbf{Y}=\lambda \mathrm{MB}_{0}$ para la pesquería de krill, en la cual la pesquería y el crecimiento del krill son estacionales. El criterio empleado para los cálculos es que la probabilidad de que la biomasa de desove del krill descienda a menos del $20 \%$ de su nivel mediano de pre-explotación, sobre un período de captura de 20 años, deberá ser igual a un $10 \%$. El valor de $\lambda$ está estrechamente ligado al valor de $\mathbf{M}$ (mortalidad natural) y de $\sigma_{\mathrm{R}}$ (variabilidad de reclutamiento), y también a los valores relativos de las edades de madurez y de primera captura. Se considera que los efectos estacionales tienen menos importancia.

## 1. INTRODUCTION

One of the topics considered by the Second Meeting of the Working Group on Krill was the estimation of potential yield (SC-CAMLR, 1990). Some members suggested that a possible approach to the determination of appropriate yields might be to use the formula:

$$
\begin{equation*}
\mathrm{Y}=\lambda \mathrm{MB}_{0} \tag{1}
\end{equation*}
$$

where $\mathbf{Y}$ is the annual yield,
$\mathbf{M}$ is the natural mortality in $\mathrm{yr}^{-1}$,
$\mathbf{B}_{0}$ is an estimate of the effective total biomass of the population prior to exploitation,
$\lambda$ is a numerical factor which is typically less than 0.5 .
A basis for calculating an appropriate value for $\lambda$ is set out in Beddington and Cooke (1983). The criterion which they use is that the probability that the spawning stock size drops below $20 \%$ of its average value prior to exploitation, over a 20 year period of harvesting, is equal to $10 \%$. Clearly the value of $\lambda$ is dependent on biological parameters such as growth rates and the extent of recruitment variability, as well as on the age-at-first capture, and Beddington and Cooke (op.cit.) provide tables giving the value of $\lambda$ for various combinations of these parameters.

However, Beddington and Cooke's calculation is based on a von Bertalanffy growth curve, which reflects steady growth throughout the year. In contrast, krill exhibits growth which varies systematically over the year (Miller and Hampton, 1989). The Second Meeting of the Working Group on Krill (SC-CAMLR, 1990) therefore suggested that, rather than use values given in Beddington and Cooke's tabulations, their calculations should be repeated for krill in a manner which takes account of seasonal growth variation, for a range of plausible values for pertinent parameters.

This paper reports the results of calculations along these lines.

## 2. METHODS

The mathematical detail of the calculations carried out is given in Appendix 1. This section comments only on the assumptions upon which those calculations are based.

The model of Rosenberg, Beddington and Basson (1986) is used to describe the seasonal growth of krill. This model for the length of krill of age $(a+t)$ years may be written:

$$
\ell(a+t)= \begin{cases}\ell_{\infty}\left[1-e^{-\beta(a+\delta t)}\right] & 0 \leq \mathrm{t} \leq 1 / \delta  \tag{2}\\ \ell_{\infty}\left[1-e^{-\beta(a+1)}\right] & 1 / \delta \leq \mathrm{t} \leq 1\end{cases}
$$

Rosenberg et al.'s (op.cit.) fit to data yields an estimate of $\delta=4$, which corresponds to a three month growth period, followed by nine months of zero growth.

Much of the fishing of krill off Antarctica is concentrated in the three summer months December to February (Miller, 1990), so that it is convenient for these calculations to assume that the three-month growth and fishing seasons coincide exactly, even though results in Rosenberg et al. (op. cit.) suggest that the krill growth season is a little earlier than this. The start of this season is taken to be the appropriate time to reflect a measure of spawning biomass.

The biomass estimate $\mathbf{B}_{0}$ which is to be used in equation (1) is considered to be made in the middle of the season (i.e. mid-January). This is an estimate of the biomass prior to any harvesting taking place, but the analysis takes account of the fact that unexploited biomass will fluctuate from year to year because of recruitment variability. Beddington and Cooke (1983) assume that the estimate is exact, but allowance for (survey sampling) estimation error has been made in this analysis [the $\eta$ term in equation (A.9) of Appendix 1]. In other respects, the method of analysis is identical to that of Beddington and Cooke (op.cit.), except that only a constant catch harvesting strategy has been considered. Beddington and Cooke also considered a constant fishing mortality strategy, but this seems inappropriate for krill. The reason is that, in practice, such a strategy could only be implemented in the form of fixed fishing effort, but for krill any simple (e.g. linear proportionality) relationship of CPUE to abundance (or equivalently fishing effort to fishing mortality) is questionable, particularly for the crude measures of effort (such as vessel-days) in terms of which regulations would have to be framed (Butterworth, 1989).

It is important to appreciate that the computations are of a "Myopic Bayes" nature (Walters, 1986). They assume that the only information available upon which to regulate the fishery for the following 20 years, is that from a single pre-exploitation biomass survey. No account is taken of the likelihood that further surveys will take place, and that their results could also be incorporated in some more complex feedback control management procedure.

The basic krill dynamics model used (see Appendix 1) assumes that natural mortality, unlike fishing mortality, has no seasonal component and takes place uniformly throughout the year. This assumption could be questioned, as krill predator abundance around Antarctica also fluctuates seasonally, because baleen whales (for example) are present primarily in the late spring to early summer months. To ascertain whether this might have a marked effect on results, some of the calculations have been repeated for the extreme assumption that all natural mortality also takes place during the three-month growth/fishing season. The appropriate equations for this alternative model (equivalent to one with no seasonal component, as investigated by Beddington and Cooke) are also given in Appendix 1.

## 3. CHOICE OF PARAMETER VALUES

The model developed in Appendix 1 has nine parameters: $\mathbf{M}, \mathbf{a}_{\mathbf{r}}, \mathbf{R}, \sigma_{\mathbf{R}}, \alpha, \beta, \mathbf{a}_{\mathbf{m}}, \mathbf{a}+$ and $\sigma_{\mathrm{s}}$. As the results of interest concern only biomass ratios [ $\mathbf{D}(\mathbf{y})$ - see Appendix 1], they are unaffected by the values chosen for R and $\alpha$, so that these two parameters can both be set equal to 1 for convenience.

Rosenberg et al. (1986) estimate $\ell_{\infty}=60 \mathrm{~mm}$ and $\beta=0.45 \mathrm{yr}^{-1}$. The values of length-at-age at the start of the season corresponding to these estimates are shown in Table 1.

Acoustic surveys of krill biomass would probably be most sensitive to krill of length 20 mm and above (D. Miller, pers. comm.), so that the lengths listed in Table 1 suggest that $\mathrm{a}^{+}=1^{+}$would be an appropriate choice. [Given a value for $\mathbf{M}$, the multiplicative factors required to modify the values of $\lambda$ reported below can be found by a simple deterministic calculation.]

It would be inappropriate to consider every possible combination of the remaining five parameters: $\mathbf{M}, \mathbf{a}_{\mathbf{r}}, \sigma_{\mathrm{R}}, \mathbf{a}_{\mathbf{m}}$ and $\sigma_{\mathrm{s}}$. The greatest uncertainty would seem to exist regarding appropriate values for $\mathbf{M}$ and $\sigma_{\mathrm{R}}$, so that a "base case" has been chosen which includes all nine combinations of the choices:

$$
\begin{array}{ll}
\mathrm{M}=0.3 ; & 0.6 ; 1.0 \quad \mathrm{yr}^{-1} \\
\sigma_{\mathrm{R}}=0.2 ; & 0.4 ; 1.0
\end{array}
$$

Fixed values $\mathrm{a}_{\mathrm{r}}=3, \mathrm{a}_{\mathrm{m}}=3$ and $\sigma_{\mathrm{s}}=0.3$ have been chosen to complete this "base case". [Some data-based case could be made for each of these choices; for example, the choice for $\sigma_{\mathrm{s}}$ seems typical of coefficients of variation reported in Anon. (1986).] The sensitivity of results to variations of these choices has been investigated only for the central element of the ( $\mathbf{M}, \sigma_{R}$ ) combinations considered, viz. ( $\mathrm{M}=0.6 \mathrm{yr}^{-1}, \sigma_{\mathrm{R}}=0.4$ ). The different choices explored are:

$$
\begin{aligned}
& \mathrm{a}_{\mathrm{r}}=2 ; 4 \\
& \mathrm{a}_{\mathrm{m}}=2 ; 4 \\
& \sigma_{\mathrm{s}}=0 ; 0.5
\end{aligned}
$$

A value $\sigma_{\mathrm{s}}=0$ is, of course, unattainable. It was selected to correspond to the calculation procedure of Beddington and Cooke (1983).

For the alternative model ( $\mathbf{M}$ concentrated in the three-month growth/fishing season), calculations have been carried out for the "base case" set of parameter value choices.

## 4. RESULTS AND DISCUSSION

Table 2 shows the estimates of $\lambda$ for the "base case" set of parameter values for the (seasonal) krill dynamics model of Appendix 1. Certain of the entries in this Table are shown as "(1.00)". For the "base case" choice of identical ages at maturity and first-capture ( $a_{m}=a_{r}=3$ ), the 2 -year-olds of the preceding year contribute to the spawning biomass (calculated at the start of the season) before the year-class concerned is affected in any way by fishing. In certain circumstances then (high $\mathbf{M}$ and low $\sigma_{R}$ ), it becomes possible to catch effectively all krill older than the age-at-first-capture, because the recruiting cohort is always large enough to satisfy the criterion that $\mathrm{B}_{\mathrm{sp}} / \mathrm{K}_{\mathrm{sp}}>0.2$. It is for such circumstances that the $\lambda$ estimate is shown as "(1.00)".

Table 3 reports estimates of $\lambda$ for the tests of sensitivity to the values chosen for $\mathbf{a}_{\mathbf{r}}, \mathbf{a}_{\mathbf{m}}$ and $\sigma_{\mathrm{s}}$. The sensitivity does not seem to be particularly marked, unless circumstances become such as lead to a "(1.00)" estimate situation.

Finally, Table 4 gives the results of assuming that $\mathbf{M}$ is concentrated in the three-month growth/fishing season (effectively equivalent to a no-seasonality situation). They show very little change from the results in Table 2.

The primary implications of these calculations are that the value of $\lambda$ is strongly dependent on the values of the $\mathbf{M}$ (natural mortality) and $\sigma_{\mathrm{R}}$ (recruitment variability) parameters, and also on the relative values of the ages at maturity and first-capture.

The seasonality of the krill fishery seems a matter of lesser importance at this stage. The calculations of this paper have assumed krill fishing over the summer season. In certain statistical areas (specifically Subarea 48.3), krill fishing actually peaks in winter (Miller, 1989). The values of $\lambda$ would obviously change for this situation. However, given the smallness of the seasonality effects evident in the Table 2 - Table 4 comparison, one would not anticipate the change in $\lambda$ for a winter fishery to be large compared to the differences in $\lambda$ evident for alternative choices for $\mathbf{M}$ and $\sigma_{\mathbf{R}}$. Clearly, efforts to narrow the range of likely values for $\mathbf{M}$ and $\sigma_{\mathbf{R}}$ merit the greatest attention at this time.

The growth equation used for krill in these calculations indicates a fairly long-lived animal. If this is correct, one would not expect a value of $\mathbf{M}$ that is very high (or too few krill would survive to the older ages indicated). Appropriate ranges for $\mathbf{M}$ for calculations such as these must therefore be correlated to some extent with the growth curve assumed.

Finally, it must be pointed out that equation (1) applies to a situation where $\mathbf{B}_{0}$ represents the total biomass of the stock concerned. In cases where a biomass survey does not cover the full range of the stock, including those where there is net transport of krill through the area surveyed, yield estimates from equation (1) will obviously contain an element of negative bias.

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Table 1: Values of krill length-at-age [ $\ell(\mathbf{a})]$ at the start of the season corresponding to the parameter estimates obtained by Rosenberg et al. (1986) for the growth model of equation (2).

| Age a <br> (years) | Length $\ell(\mathbf{a})$ <br> $(\mathbf{m m})$ |
| :---: | :---: |
| 0 | 0 |
| 1 | 22 |
| 2 | 36 |
| 3 | 44 |
| 4 | 50 |
| 5 | 54 |
| 6 | 56 |
| 7 | 57 |

Table 2: Estimates of the harvesting fraction parameter $\lambda$ (where $\mathrm{Y}=\lambda \mathrm{MB}_{\text {obs }}$ ) for the krill dynamics model of Appendix 1 with the "base case" set of parameter value choices: $\mathrm{a}_{\mathrm{r}}=3 ; \mathrm{a}_{\mathrm{m}}=3$ and $\sigma_{\mathrm{s}}=0.3$.

|  | $\mathrm{M}=0.3 \mathrm{yr}^{-1}$ | $\mathrm{M}=0.6 \mathrm{yr}^{-1}$ | $\mathrm{M}=1.0 \mathrm{yr}^{-1}$ |
| :--- | :---: | :---: | :---: |
| $\sigma_{\mathrm{P}}=0.2$ | 0.43 | $(1.00)$ | $(1.00)$ |
| $\sigma_{\mathrm{P}}=0.4$ | 0.39 | 0.23 | 0.21 |
| $\sigma_{\mathrm{P}}=1.0$ | 0.20 | 0.08 | 0.04 |

Table 3: Tests of the sensitivity of estimates of $\lambda$ for the case $\left(M=0.6 \mathrm{yr}^{-1}, \sigma_{\mathrm{R}}=0.4\right)$ to variations in the "base case" (BC) choices for parameters $\mathbf{a}_{\mathrm{r}}, \mathbf{a}_{\mathbf{m}}$ and $\sigma_{\mathrm{s}}$.

| Parameter Change | $\lambda$ |
| :---: | :---: |
| BC | 0.23 |
| $\mathrm{a}_{\mathrm{x}}=2$ | 0.18 |
| $\mathrm{a}_{\mathrm{r}}=4$ | $(1.00)$ |
| $\mathrm{a}_{\mathrm{m}}=2$ | $(1.00)$ |
| $\mathrm{a}_{\mathrm{m}}=4$ | 0.15 |
| $\sigma_{\mathrm{s}}=0$ | 0.25 |
| $\sigma_{\mathrm{s}}=0.5$ | 0.20 |

Table 4: Estimates of $\lambda$ for the alternative krill dynamics model ( $\mathbf{M}$ concentrated in the three-month growth and fishing season) for the "base case" set of parameter value choices.

|  | $\mathrm{M}=0.3 \mathrm{yr}^{-1}$ | $\mathrm{M}=0.6 \mathrm{yr}^{-1}$ | $\mathrm{M}=1.0 \mathrm{yr}^{-1}$ |
| :--- | :---: | :---: | :---: |
| $\sigma_{P}=0.2$ | 0.44 | $(1.00)$ | $(1.00)$ |
| $\sigma_{P}=0.4$ | 0.40 | 0.24 | 0.25 |
| $\sigma_{P}=1.0$ | 0.20 | 0.08 | 0.04 |

## MATHEMATICAL DETAILS OF PROCEDURE USED TO CALCULATE $\boldsymbol{\lambda}$

## 1. KRILL DYNAMICS MODEL

The numbers of krill of age a years at the start of "year" $\mathbf{y}\left(\mathbf{N}_{\mathbf{y}, \mathrm{a}}\right)$ are governed by the equations:

$$
N_{y+1, a+1}= \begin{cases}N_{y, a} e^{-M} & 0 \leq a<a_{r}  \tag{A.1}\\ N_{y, a} e^{-[M+F(y)]} & a_{r} \leq a \leq 7\end{cases}
$$

where "year" is a 12 -month period whose commencement coincides with the start of the season during which krill grow,
$\mathbf{M}$ is the natural mortality rate (effective throughout the year),
$\mathbf{F}(\mathbf{y})$ is an effective annual fishing mortality rate in year $\mathbf{y}$, and
$\mathbf{a}_{\mathbf{r}} \quad$ is the (knife-edge) age-at-first-capture.
These equations assume that the number of krill 8 years of age and older is relatively small and so can be neglected.

The number of krill recruits at the start of each year $\left(\mathbf{N}_{\mathrm{y}, 0}\right)$ is given by:

$$
\begin{equation*}
\mathrm{N}_{\mathrm{y}, 0}=\mathrm{R} \exp \left(\varepsilon_{\mathrm{y}}-\sigma_{\mathrm{R}}^{2} / 2\right) \quad \varepsilon_{\mathrm{y}} \text { from } \mathrm{N}\left(0 ; \sigma_{\mathrm{R}}^{2}\right) \tag{A.2}
\end{equation*}
$$

where $\mathbf{R} \quad$ is the mean recruitment, and
$\mathbf{N}\left(0 ; \sigma^{2}\right)$ is a normal distribution with zero mean and variance $\sigma^{2}$.
Mass-at-age is obtained from the growth curve form used by Rosenberg et al. (1986), together with the assumption that mass is proportional to length cubed. Thus the mass of krill of age a at the start of the "year" $\left(w_{a}\right)$ is given by:
$\mathrm{w}_{\mathrm{a}}=\alpha\left[1-\mathrm{e}^{-\beta \mathrm{a}}\right]^{3}$
In the middle of the season during which the krill grow, their mass-at-age ( $\overline{\mathrm{w}}_{\mathrm{a}}$ ) is then:

$$
\begin{equation*}
\overline{\mathrm{w}}_{\mathrm{a}}=\alpha\left[1-\mathrm{e}^{-\beta(\mathrm{a}+0.5)}\right]^{3} \tag{A.4}
\end{equation*}
$$

The spawning biomass of krill (taken to correspond to the start of the "year") is accordingly:

$$
\begin{equation*}
\mathrm{B}_{\mathrm{sp}}(\mathrm{y})=\sum_{\mathrm{a}=\mathrm{a}_{\mathrm{m}}}^{7} \mathrm{w}_{\mathrm{a}} \mathrm{~N}_{\mathrm{y}, \mathrm{a}} \tag{A.5}
\end{equation*}
$$

where $\mathbf{a}_{\mathrm{m}}$ is the age-at-maturity for krill.

The median spawning biomass prior to any exploitation $\left(\mathbf{K}_{\mathrm{sp}}\right)$ is therefore:

$$
\begin{equation*}
K_{s p}=\sum_{a=a_{m}}^{7} w_{a} R \exp \left(-\sigma_{R}^{2} / 2\right) \mathrm{e}^{-\mathrm{Ma}}=\mathrm{R} \exp \left(-\sigma_{\mathrm{R}}^{2} / 2\right) \alpha \sum_{\mathrm{a}=\mathrm{a}_{\mathrm{m}}}^{7} \mathrm{e}^{-\mathrm{Ma}}\left[1-\mathrm{e}^{-\beta \mathrm{a}}\right]^{3} \tag{A.6}
\end{equation*}
$$

Assuming further that the growth and fishing seasons coincide, and are of three months duration, the mid-season biomass of krill aged a years and above $\left(\mathbf{B}_{\mathrm{a}+}\right)$ is given by:

$$
\begin{equation*}
\mathrm{B}_{\mathrm{a}+}(\mathrm{y})=\sum_{\mathrm{a}=\mathrm{a}+}^{7} \overline{\mathrm{w}}_{\mathrm{a}} \mathrm{~N}_{\mathrm{y}, \mathrm{a}} \exp [-\{1.5 \mathrm{M} / 12+\mathrm{F}(\mathrm{y}) / 2\}] \tag{A.7}
\end{equation*}
$$

The catch in number of krill over a season from a single cohort of size $\mathbf{N}_{\mathbf{0}}$ at the start of the "year" will be given by an equation of the form:

$$
\int_{0}^{0.25} 4 F \bullet N(t) d t=\int_{0}^{0.25} 4 F \bullet N_{0} e^{-(M+4 F) t} d t
$$

Making allowance for krill mass and its growth during the season, the annual catch in mass [ $\mathbf{C}(\mathbf{y})$ ] is therefore given by:

$$
\begin{align*}
\mathrm{C}(\mathrm{y}) & =\sum_{a=a_{r}}^{7} \int_{0}^{0.25} 4 F(y) \cdot N_{y, a} e^{-[M+4 F(y)] t} \alpha\left[1-e^{-\beta(a+4 t)}\right]^{3} d t= \\
& =4 F(y) \alpha \sum_{a=a_{r}}^{7} N_{y, a} \sum_{\mu=0}^{3} \gamma(\mu) e^{-\mu \beta a}\left[1-e^{-\{M+4 F(y)+4 \mu \beta\} / 4}\right] /[M+4 F(y)+4 \mu \beta] \tag{A.8}
\end{align*}
$$

where $\gamma(0)=1 ; \gamma(1)=-3 ; \gamma(2)=3$ and $\gamma(3)=-1$.

## 2. THE ALGORITHIM FOR THE COMPUTATIONS

The results required are computed using the following algorithm.
(i) For given values of biological parameters and the age-at-first-capture, choose a value for $\lambda$.
(ii) For "year" $\mathrm{y}=1$, set up deterministic equilibrium age-structure for $\mathrm{F}=0$, i.e.:

$$
\mathrm{N}_{1, \mathrm{a}}=\mathrm{R} \exp \left(-\sigma_{\mathrm{R}}^{2} / 2\right) \mathrm{e}^{-\mathrm{Ma}} \quad \mathrm{a}=0,1, \ldots, 7
$$

Note that since results are required for biomass ratios only, $\mathbf{R}$ can be set equal to 1 .
(iii) Project the numbers-at-age forward in the absence of exploitation $[\mathrm{F}(\mathrm{y})=0]$ using equations (A.1), with stochastic recruitment given by equation (A.2), for "years" $\mathrm{y}=2$ to 11 . Thus, by "year" $\mathrm{y}=10$, all memory of the initial deterministic age-structure has been lost.
(iv) In year $\mathrm{y}=10$ (the year before fishing commences), the $\mathrm{a}+$ biomass is "measured" in mid-season, yielding a result $\mathbf{B}_{\text {obs }}$ where:

$$
\begin{equation*}
B_{o b s}=\sum_{a=a^{+}}^{7} \bar{w}_{a} N_{10, a} \exp [-1.5 M / 12] \exp \left[\eta-\sigma_{s}^{2} / 2\right] \tag{A.9}
\end{equation*}
$$

where $\eta$ from $\mathrm{N}\left(0 ; \sigma_{\mathrm{s}}^{2}\right)$ is the log normal variation associated with the biomass survey.
(v) Fix a constant catch $\mathrm{C}=\lambda \mathrm{MB}_{\text {obs }}$.
(vi) For year $\mathrm{y}=11$, find the fishing mortality $\mathbf{F}(\mathrm{y})$ which satisfies equation (A.8) for the catch computed in v). [To speed computations, in instances where $F(y)$ exceeds 5 , it is set to $\infty$, i.e. all the krill above the age-at-first-capture are harvested.] Project the age-structure forward to $y=12$ using that value of $\mathbf{F}(\mathbf{y})$ in equations (A.1). Calculate the ratio of the spawning biomass to its median pre-exploitation level, $\mathrm{D}(\mathrm{y})=\mathrm{B}_{\mathrm{sp}}(\mathrm{y}) / \mathrm{K}_{\text {sp }}$ for $\mathrm{y}=12$. [Note that $\mathrm{y}=12$ is the first year for which the spawning biomass has been influenced by the effects of fishing.]
(vii) Repeat step (vi) for $\mathrm{y}=12$ to 30 , thereby obtaining values of $\mathbf{D}(\mathrm{y})$ for $\mathrm{y}=13$ to 31 .
(viii) Ascertain whether any elements of the set $\{\mathbf{D}(\mathbf{y}): \mathrm{y}=12, \ldots 31\}$ are less than 0.2.
(ix) Repeat steps (iii) to (viii) a large number of times ( 1000 simulations was found to provide adequate precision for this study), to estimate the probability $\mathbf{P}$ that a value of $\mathbf{D}(\mathbf{y})$ less than 0.2 occurs during the 20 year harvesting period.
(x) Repeat steps (ii)) to (ix) for different values of $\lambda$ until one is found which yields the result $P=0.1$. This $\lambda$ value is the result required. [For reasons explained in the text, an upper bound of 1.0 is set for $\lambda$ in this exercise.]

Note that to ensure a smoother relationship between $\lambda$ and $\mathbf{P}$ so that computations [specifically step (x)] are eased, the same set of random numbers [ $\eta$ and $\varepsilon(y)$ ] are used for different values of $\lambda$.

## 3. ALTERNATIVE DYNAMICS MODEL

Under the extreme assumption that all natural mortality also takes place during the three-month growth and fishing season, the problem becomes equivalent to the standard form where seasonality has no effect. The consequent modifications to equations (A.6), (A.8) and (A.9) involve effectively the replacement $\mathrm{M} \rightarrow 4 \mathrm{M}$, which yields respectively:

$$
\begin{align*}
& B_{a^{+}}(y)=\sum_{a=a^{+}}^{7} \bar{w}_{a} N_{y, a} \exp [-\{M+F(y)\} / 2]  \tag{A.10}\\
& C(y)=F(y) \alpha \sum_{\mathrm{a}=\mathrm{a}_{\mathrm{r}}}^{7} N_{y, a} \sum_{\mu=0}^{3} \gamma(\mu) e^{-\mu \beta a}\left[1-e^{\{M+F(y)+\mu \beta\}}\right] /[M+F(y)+\mu \beta]  \tag{A.11}\\
& B_{o b s}=\sum_{a=a^{+}}^{7} \bar{w}_{a} N_{10, a} \exp [-M / 2] \exp \left[\eta-\sigma_{s}^{2} / 2\right] \tag{A.12}
\end{align*}
$$

## Liste des tableaux

Tableau 1: Les valeurs des longueurs du krill selon l'âge [ $\ell(\mathbf{a})$ ] au début de la saison, correspondant aux estimations des paramètres obtenus par Rosenberg et al. (1986) pour le modèle de croissance de l'équation (2).

Tableau 2: $\quad$ Estimations du paramètre $\lambda$ de la fraction disponible pour l'exploitation (lorsque $\mathrm{Y}=\lambda \mathrm{MB}_{\text {obs }}$ ) dans le modèle de dynamique du krill de l'appendice 1 avec comme "repères" les valeurs suivantes des paramètres: $a_{r}=3 ; a_{m}=3$ et $\sigma_{s}=0,3$.

Tableau 3: $\quad$ Tests de sensibilité des estimations de $\lambda$ dans le cas où ( $M=0,6 \mathrm{an}^{-1}, \sigma_{R}=0,4$ ) aux variations des choix des "repères" (BC) des paramètres $\mathbf{a}_{\mathrm{r}}, \mathbf{a}_{\mathrm{m}}$ et $\sigma_{\mathbf{s}}$.

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## Список таблиц

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Таблица 3: Чувствительность оценок $\lambda$, когда ( $\mathrm{M}=0.6 \mathrm{yr}^{-1}, \sigma_{\mathrm{R}}=0.4$ ), к изменениям исходного набора (BC) альтернативных параметров $\mathbf{a}_{\mathbf{r}}$, $\mathbf{a}_{\mathrm{m}}$ и $\sigma_{\mathrm{s}}$.

Таблица 4: Оценки $\lambda$ для альтернативной модели динамики криля (М сосредоточено в трехмесячном сезоне роста и промысла) для исходного набора альтернативных параметров.

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