

## Multi-fleet stock assessment modelling with the Grym

J. Liu<sup>1,2</sup>, S. Wotherspoon<sup>1,3</sup>, D. Maschette<sup>1,3</sup>

<sup>1</sup> Fisheries and Aquaculture Centre, Institute for Marine and Antarctic Studies  
University of Tasmania  
Hobart, Tasmania 7001  
Australia  
Email contact: [dale.maschette@aad.gov.au](mailto:dale.maschette@aad.gov.au)

<sup>2</sup> College of Oceanic and Atmospheric Sciences  
Ocean University of China  
5 Yushan Road, Shinan District, Qingdao  
Shandong 266005  
China

<sup>3</sup> Department of Climate Change, Energy, the Environment and Water  
Australian Antarctic Division  
203 Channel Highway, Kingston, Tasmania 7050  
Australia

### Abstract

Grym is an open-source implementation of the generalised yield model in R that provides greater transparency and extensibility (Maschette et al., 2023; Wotherspoon and Maschette, 2020). This paper describes an extension of the Grym to permit modelling of multiple fleets within a season, allowing the Grym implementation to model more complex fishery behavior and evolving fisheries practices. Results from the extension are compared with existing analyses from the Grym. The model can include vessels using different or identical selectivities with an example presented for Patagonian toothfish.

### Résumé

Le Grym est une exécution du modèle de rendement généralisé dans R en libre accès permettant plus de transparence et d'extensibilité (Maschette *et al.*, 2023 ; Wotherspoon et Maschette, 2020). La présente étude décrit une extension du Grym grâce à laquelle il est possible de modéliser plusieurs flottilles lors d'une même saison, permettant à l'exécution du Grym de modéliser un comportement de la pêche plus complexe et des pratiques de pêche changeantes. Les résultats issus de cette extension sont comparés aux analyses du Grym existantes. Le modèle peut inclure des navires utilisant des sélectivités différentes ou identiques, avec un exemple présenté pour la légine australe.

### Абстракт

Grym – это версия обобщенной модели вылова на языке R с открытым исходным кодом, обеспечивающая больше прозрачности и возможности расширения (Maschette et al., 2023; Wotherspoon and Maschette, 2020). В этом документе описано расширение для Grym, позволяющее моделировать несколько промысловых флотилий в течение сезона, что дает возможность Grym моделировать более сложное промысловое поведение и меняющуюся практику ведения промысла. Результаты, полученные с помощью расширения, сравниваются с существующими аналитическими данными Grym. Модель позволяет включать суда, использующие различную или идентичную селективность; пример представлен для патагонского клыкача.

## Resumen

Grym es una implementación en código abierto del modelo de rendimiento generalizado en R que aporta una mayor transparencia y extensibilidad (Maschette et al., 2023; Wotherspoon and Maschette, 2020). El presente documento describe una extensión del Grym para poder modelar múltiples flotas dentro de una temporada, permitiendo así a esta implementación del Grym modelar comportamientos más complejos de la pesquería y prácticas de pesca cambiantes. Los resultados de esta extensión son comparados con análisis Grym anteriores. El modelo puede incluir barcos con selectividades idénticas o diferentes. El documento contiene un ejemplo de ello para el que se utiliza la austrómerluza negra.

## Introduction

Recently, the generalised yield model (GYM) was re-implemented in the open-source R software as the ‘Grym’ package to improve computer compatibility and model transparency (Wotherspoon and Maschette, 2020; Maschette et al., 2023). The basic purpose of the Grym is to provide an open-source implementation of the GYM software (Constable and de la Mare, 1996) that could be used as a base package to support a range of population assessment and management strategy applications (Maschette et al., 2023).

The GYM and the current version of the Grym assume a single fishing fleet in each season, represented though a single fishing selectivity. This study addresses a long-standing request from SC-CAMLR in 2003 to extend the GYM assessment to multiple fleets (SC-CAMLR-XXII, Annex 5, paragraph 9.6 (viii)). A series of functions, designed to extend the functionality of the Grym to allow multiple fleets, are presented to achieve this.

This paper first describes and compares the methods and mathematical formulas used in the GYM and Grym models to predict stock abundance, biomass and yield. Based on these formulas, new formulas and methods generated to extend the prediction functions to allow multiple fleets have been expanded and added. The application of these methods is shown with new multi-fleet functions in R. A series of example tests is also provided to demonstrate the use of these functions and compare to single fleet Grym fits for a Patagonian toothfish example. Finally, we give a detailed description and code of the multi fleet functions developed and the tests carried out (Appendix 1).

## The basic population model

In both implementations (the Grym or GYM), models are an age-structured cohort model, and use numerical integration to calculate the annual change of stock abundance, biomass and the yield for each cohort over a single year. Iterating this process allows the stock projections forward in time.

Within each year the abundance in an age class is assumed to satisfy the Baranov catch equation (Xiao, 2005)

$$\frac{dN_a}{dt} = -(M(a,t) + F(a,t))N_a \quad (1)$$

Here  $N_a(t)$  is the abundance in age class  $a$  at time  $t$  within the year, and  $M(a,t)$  and  $F(a,t)$  are the instantaneous natural and fishing mortalities experienced by an individual in age class  $a$  at time  $t$ . The natural mortality is represented as the product

$$M(a,t) = M_y m(a,t) \quad (2)$$

where  $m(a,t)$  describes the pattern of intra-annual variation in mortality for each age class, and  $M_y$  is an annual scaling that can be randomly varied to represent any year-to-year variability in natural mortality. Similarly, fishing mortality is represented as the product

$$F(a,t) = F_y E(t) S(a,t) \quad (3)$$

where  $E(t)$  represents the distribution of fishing effort over the year,  $S(a,t)$  the age-dependent gear selectivity, and  $F_y$  is an overall annual scaling of fishing effort.

Both implementations assume a strict deterministic relationship between weight and age. The

weight  $W_a(t)$  of an individual in age class  $a$  at time  $t$  must be specified, and this factor is used to convert the abundance in each age class to the corresponding biomass

$$B_a(t) = w_a(t)N_a(t) \quad (4)$$

and the total biomass is determined by summing the contributions from each age class

$$B(t) = \sum_a B_a(t) \quad (5)$$

Similarly, the yield in each age class is the number of harvested individuals re-expressed as biomass

$$\frac{dY_a}{dt} = w_a(t)F(a,t)N_a \quad (6)$$

and the total yield  $Y(t)$  is then determined by summing the contributions from each age class

$$Y(t) = \sum_a Y_a(t). \quad (7)$$

At the end of each year, surviving individuals advance to the next highest age class, and the youngest age class is filled with new recruits generated randomly according to an assumed recruitment model.

### Model integration

The original implementation of the GYM (Constable and de la Mare, 1996) integrated the governing equations (1) to (7) with a standard Runge Kutta method (Burden and Faires, 2005). The Grym implementation, however, takes a more direct approach (Maschette et al., 2023). The governing equations may be solved directly to yield

$$N_a(t) = N_a(0) \exp\left(-M_y \int_0^t m(a,\tau)d\tau - F_y \int_0^t E(\tau)S(a,\tau)d\tau\right) \quad (8)$$

$$B_a(t) = w_a(t)N_a(t) \quad (9)$$

$$Y_a(t) = F_y \int_0^t E(\tau)S(a,\tau)B_a(\tau)d\tau \quad (10)$$

where the integrals in (8) and (10) are evaluated with the composite trapezoidal rule (Burden and Faires, 2005). If the intra-annual mortalities  $m(a,t)$  and  $f(a,t)$  are assumed piecewise linear, this yields exact solutions for the abundance  $N_a(t)$  and biomass  $B_a(t)$ .

Equations (8) to (10) provide the abundance, biomass and yield for given initial abundances  $N_a(0)$ . But as the governing equations are linear, any rescaling of a solution is also a valid solution. Rescaling  $N_a(t)$ ,  $B_a(t)$  and  $Y_a(t)$  by the factor

$$r = \frac{B_{ref}(t_1 - t_0)}{\sum_a \int_{t_0}^{t_1} B_a(\tau)d\tau} \quad (11)$$

generates an annual projection that has initial relative abundances  $N_a(0)$  and mean total biomass  $B_{ref}$  in the time interval  $[t_0, t_1]$ , allowing the projection to be scaled to a total biomass estimate from a survey.

Typically, the annual scaling  $F_y$  is not known *a priori* and is determined by matching the total annual yield  $Y(1)$  to a known historic or target catch. When there is only a single fleet,  $F_y$  can be determined through an appropriate univariate root finding method. Both model implementations use Brent's method (Burden and Faires, 2005) for this purpose.

### Extension to multiple fleets

The single fleet model described above generalises naturally to a multiple fleet model in which each fleet makes a separate contribution to the total mortality. However, the task of determining the annual effort required to reproduce known historic or target catches is much more complex in the multiple fleet case.

To generalise the Grym to multiple fleets, the total fishing mortality is decomposed into contributions from each fleet

$$\begin{aligned}
F(a, t) &= \sum_k F_k(a, t) \\
&= \sum_k F_{k,y} E_k(t) S_k(a, t). \quad (12)
\end{aligned}$$

Here  $F_k(a, t)$  is the contribution to the total fishing mortality from fleet  $k$ , and this is further decomposed into a distribution  $E_k(t)$  of fishing effort over the year, an age-dependent gear selectivity  $S_k(a, t)$  and the annual scaling  $F_{k,y}$  of fishing effort for that fleet, as the fishing season, gear selectivity and overall investment of effort will vary from fleet to fleet.

With these definitions, the Grym's governing equations become

$$\frac{dN_a}{dt} = - \left( M(a, t) + \sum_k F_k(a, t) \right) N_a. \quad (13)$$

$$B_a(t) = w_a(t) N_a(t) \quad (14)$$

$$\frac{dY_{k,a}}{dt} = w_a(t) F_k(a, t) N_a \quad (15)$$

where the yield is now calculated for each fleet and age class, and the total yield per fleet is determined by summing contributions from each age class

$$Y_k(t) = \sum_a Y_{k,a}(t). \quad (16)$$

In turn these governing equations have the solution

$$\begin{aligned}
N_a(t) &= N_a(0) \\
&\exp \left( -M_y \int_0^t m(a, \tau) d\tau - \right. \\
&\quad \left. \sum_k F_{k,y} \int_0^t E_k(\tau) S_k(a, \tau) d\tau \right) \quad (17)
\end{aligned}$$

$$B_a(t) = w_a(t) N_a(t) \quad (18)$$

$$Y_{k,a}(t) = F_{k,y} \int_0^t E_k(\tau) S_k(a, \tau) B_a(\tau) d\tau \quad (19)$$

where again the necessary integrals can be evaluated with the composite trapezoidal rule.

As in the case of a single fleet, equations (16) and (17) determine the abundance, biomass and

yield for known initial abundances  $N_a(0)$ . If only relative initial abundances are known, then rescaling  $N_a(t)$ ,  $B_a(t)$  and  $Y_a(t)$  by the factor

$$r = \frac{B_{ref}(t_1 - t_0)}{\sum_a \int_{t_0}^{t_1} B_a(\tau) d\tau} \quad (20)$$

again scales the projection to a mean total biomass estimate  $B_{ref}$  from a survey taken over the time interval  $[t_0, t_1]$ .

When the annual scaling  $F_{k,y}$  of fishing effort is known, projecting stock abundance, biomass and yield forward over a year for multiple fleets model follows essentially the same process as for the single fleet case, except that yield is calculated on a per fleet basis, and total fishing mortality consists of a contribution from each fleet.

When there are multiple fleets, however, determining the annual scaling  $F_{k,y}$  that matches the annual yield for each fleet to known historic or target catches is more difficult. In the single fleet case, only a single  $F_y$  needs to be determined in each year, and any reliable univariate root finding methods can be employed. In the multiple fleet case, a scaling  $F_{k,y}$  must be determined for each fleet, but these scalings are not independent. Although the individual fleets operate independently, increasing the effort of one fleet reduces the available biomass, making it more difficult for the other fleets to attain their targets.

When the simulated stock is highly depleted, the target catch for some or all fleets may be unattainable, as there is insufficient biomass in the population for some fleets to catch their target. With a single fleet, this event is easily detected and the entirety of the attainable yield can be attributed to the only fleet. When there are multiple fleets however, the attainable yield must be correctly apportioned amongst each of the fleets.

When there are multiple fleets, the annual scaling  $F_{k,y}$  that matches the annual yield to a given target can be determined through modified form of Newton's method (Burden and Faires, 2005). Let  $F = (F_1, F_2, \dots, F_m)$  denote the vector of scalings for the  $m$  fleets fishing in the current year,

$Y(F) = (Y_1(F), Y_2(F), \dots, Y_m(F))$  the vector of yields for the  $m$  fleets, and  $C = (C_1, C_2, \dots, C_m)$  the corresponding vector of target catches. Then the required scalings are the root of the equation:

$$Y(F) - C = 0. \quad (21)$$

Newton's method generates a sequence of successively better approximations  $F^{(1)}, F^{(2)}, \dots$  to the root of this equation according to the recursion

$$F^{(n+1)} = F^{(n)} - J^{-1}(Y - C) \quad (22)$$

where  $J$  is the Jacobian matrix with elements:

$$J_{ij} = \frac{\partial Y_i}{\partial F_j}. \quad (23)$$

When the projection is not rescaled to match a reference biomass, the elements of the Jacobian matrix can be determined directly from equations (16) to (18). Writing  $f_k(a, t) = E_k(t)S_k(a, t)$  and suppressing subscripts in  $y$  for clarity, differentiating equations (16) to (18) with respect to  $F_j$  shows that:

$$\begin{aligned} \frac{\partial N_a}{\partial F_j} = & -N_a(0) \int_0^t f_j(a, \tau) d\tau \exp \\ & \left( -M_y \int_0^t m(a, \tau) d\tau - \sum_k F_k \int_0^t f_k(a, \tau) d\tau \right) \end{aligned} \quad (24)$$

$$\frac{\partial B_a}{\partial F_j} = w_a(t) \frac{\partial N_a}{\partial F_j} \quad (25)$$

and hence

$$\begin{aligned} \frac{\partial Y_{i,a}}{\partial F_j} = & \delta_{ij} \int_0^t f_i(a, \tau) B_a(\tau) d\tau - \\ & \int_0^t F_i f_i(a, \tau) B_a(\tau) \int_0^\tau f_i(a, \sigma) d\sigma d\tau \end{aligned} \quad (26)$$

where  $\delta_{ij}$  denotes the Kronecker delta and again the required integrals are evaluated with the composite trapezoidal rule.

When the projection is scaled to match a reference biomass, the yield  $Y_{k,a}(t)$  is scaled by the additional factor  $r$  from equation (20), and it can be shown that:

$$\begin{aligned} \frac{\partial Y_{i,a}}{\partial F_j} = & r \left( \delta_{ij} \int_0^t f_i(a, \tau) B_a(\tau) d\tau - \right. \\ & \left. \int_0^t F_i f_i(a, \tau) B_a(\tau) \int_0^\tau f_i(a, \sigma) d\sigma d\tau \right) + \\ & \frac{\sum_a \int_{t_0}^{t_1} \int_0^\tau f_j(a, \sigma) d\sigma B_a(\tau) d\tau}{\sum_a \int_{t_0}^{t_1} B_a(\tau) d\tau} \\ & \int_0^t F_i f_i(a, \tau) B_a(\tau) d\tau. \end{aligned} \quad (27)$$

In both cases the entries of the Jacobian matrix are the sums of these derivatives over the age classes

$$J_{ij} = \frac{\partial Y_i}{\partial F_j} = \sum_a \frac{\partial Y_{i,a}}{\partial F_j}. \quad (28)$$

The Newton iteration (20) is initialised with  $F^{(1)} = \mathbf{0}$  and is terminated when the projected yield  $Y(F^{(n+1)})$  reproduces the target  $C$  to within an appropriate tolerance.

When the target is unattainable for one or more fleets, the scaling for that fleet increases with each Newton iteration as the method tries to drive the yield towards the target. But for very large  $F$ , the trapezoidal rule provides a poor approximation to the yield and the projection may become unreliable. To avoid this, each  $F_k$  is capped at some maximum  $F_{\max}$  that corresponds to a fishing effort that would be impossible to deliver. In each Newton iteration, the fleet for which the scaling  $F_k$  exceeds  $F_{\max}$  and yet the annual yield  $Y_k(1)$  remains below the target  $C_k$ , the scaling  $F_k$  is fixed at  $F_{\max}$  and the Newton iteration recalculated for the remaining fleets. The iteration is terminated when for each fleet, either  $F_k = F_{\max}$  or the projected yield  $Y_k(F^{(n+1)})$  reproduces the target  $C_k$  to within an appropriate tolerance. This process is summarised in Figure 1.

## Examples and tests

A number of examples were conducted to test the basic functionality of the multi-fleet expansion. These models are based upon fishing fleets which fish continuously through their respective season until either their allocated catch is reached, or the fleet hits the maximum effort allowed within the model.

The first scenario compared the multiple fleet implementation to the original single fleet implementation presented in Maschette et al. (2023). In the simplest form, two identical fleets, with identical fishing seasons, selectivities and target catches, should provide similar results to a single fleet model. As the two fleets are identical, each fleet should have identical yields and annual scaling of fishing effort. If the catch is unattainable, the attainable catch will be equally apportioned between the two fleets and the annuals scaling should be capped at its maximum.

When comparing fleets with the same fishing season to sequential season fleets, the models behave largely as expected (Table 3). When two fleets fish at the same time, each obtains the same catch with the same effort. In contrast, when one fleet fishes after the other, the second fleet requires more effort to obtain the same catch, or if there is not enough biomass in the population, hits the  $f_{max}$  of the model and receives less catch.

## Comparisons

A comparison was made using the toothfish assessment fitted as a single fleet model and simulated for 10001 runs presented in Maschette et al. (2023). This model was compared to: 1) the same parameters fitted as a multi-fleet model where both fleets have the same selectivity as the single-fleet model, and 2) a multi-fleet model where the two fleets used a differing double normal selectivity, simulating a trawl and longline fleet (Figure 2). These were compared using two diagnostics: 1) the probability of depletion; the probability of the stock dropping below 20% SSB ( $S_{min} < 0.2S_0$ ), and 2) the mean spawning stock escapement ( $\hat{E}$ ).

The model projections for the two example cases performed as expected (Table 4). For the case of two fleets with identical selectivities, the two fleets show identical performances, and the multi-fleet projections agree with the projections from an equivalent single fleet model. For the model with two fleets that fish in sequence, the fleet that fishes earlier in the season obtains its target catch with less effort than the fleet that fishes later in the season. When the overall target is unattainable, for lower targets the early fleet still attains its target while the later fleet does not, and at higher targets neither fleet attains its target.

## Discussion

This study has presented an extension of the Grym stock projection model to allow for multiple fishing fleets. The basic Grym model has been extended to perform stock projections when the fishing mortality due to each fleet is known, and a modified form of Newton's method is developed for determining the annual scaling of fishing effort required to reproduce a known target catch for each fleet. If the target catches are unattainable, the attainable component of the catch is correctly apportioned amongst the fleets.

This work has demonstrated that the extension of Grym's 'project' functions to multiple fleets produces outputs that are very consistent with the results from Maschette et al. (2023) for the Patagonian toothfish fishery assessment when selectivity is the same. When selectivity is different, the results are consistent with what may be expected.

Beyond the ability to better reflect fishing dynamics in an assessment when two fleets are present, the multi-fleet functionality also presents the opportunity to be used in other ways. One such way may be the inclusion of predator classes as a 'fleet' in order to be able to test assumptions around a population's ability to deal with the combination of fishing and predation. This allows for the Grym to be used in a much larger range of applications than a single fleet model alone can achieve.

Given the capacity to model more complex fisheries, fishing practices, and the demonstrated high degree of consistency in the outputs, the expansion presented in this study provides greater flexibility and options when performing stock assessments or management strategy evaluations with the Grym.

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Table 1: Newly added Grym package functions with brief descriptions.

Functions	Description
projectMF	Function that projects the abundance, biomass and yield in each age class forward over one year, allowing for multiple fleets
projectCMF	Function to project the abundance, biomass and yield in each age class forward over one year, given a known target catch for each fleet
scale3	The function that scales a 3D array along its third dimension or optionally summing along the third dimension
trapz, ctrapz,	Numerical quadrature by the composite trapezoidal rule for 3D arrays

Table 2: Input parameters for Grym assessment comparisons conducted for *Dissostichus eleginoides*.

Category	Parameter	Value
Ages	First age class	4
	Last age Class	35
von Bertalanffy growth	$t_0$	0
	$L_\infty$	170.8 cm
	$k$	0.088
	Date – start growth period (dd/mm)	30-Nov
	Date – end growth period (dd/mm)	30-Nov
Weight at length (kg, mm)	Weight-length parameter – A (kg)	2.50E-05
	Weight-length parameter – B	2.8
Maturity	Age, 50% are mature	9.81
	Range over which maturity occurs	12
	Years over which maturity occurs	Jan-17
Spawning season	First day of spawning Season (dd/mm)	1-Jul
	Last day of spawning Season (dd/mm)	1-Jul
Mortality	Min. mean annual $M$	0.16
	Max. mean annual $M$	0.16
Recruitment	Function	Log-Normal
	Mean recruitment	14.6372
	Coefficient of variation	1.162
Fishery parameters	Length 50% selected	70 cm
	Range over selection	10 cm
	Season	1 Dec – 30 Nov
	Catch between survey and season	0
Simulation specifications	Number of runs in simulation	10001
	Evaluation type	Constant
		Catch
Individual trial specifications	Years to remove initial age structure	1
	Reference start date in year	1-Dec
	Increments in year	360
	Years to project stock in simulation	35
	Reasonable upper bound for annual F	4.5
	Tolerance for finding F in each year	0.000001

Table 3: Comparison of single runs of Patagonian toothfish models for identical and sequential fleets on the multiple fleet implementation. Catch is reported in grams to allow comparison. Effort is annual fishing mortality, to a maximum of 4.5.

Target	Catch		Effort		
	Fleet 1	Fleet 2	Fleet 1	Fleet 2	
Identical fleets	2.0E8	100,000,000	100,000,000	0.2809625	0.2809625
	3.0E8	150,000,000	150,000,000	0.5140667	0.5140667
	4.0E8	200,000,000	200,000,000	0.9536835	0.9536835
	5.0E8	237,273,831	237,273,831	4.5	4.5
		Fleet 1	Fleet 2	Fleet 1	Fleet 2
Sequential fleets	2.0E8	100,000,000	100,000,000	1.129263	2.440313
	3.0E8	150,000,000	82,284,240	2.059255	4.5
	4.0E8	200,000,000	37,423,276	3.776633	4.5
	5.0E8	211,519,365	27,058,807	4.5	4.5



Table 4: Comparison of single and multiple fleet assessment models with 10001 projection runs of six long-term annual yields for *Dissostichus eleginoides*. Single denotes the single fleet fit of the Grym, Multiple 1 is two fleets fishing with the same selectivity, and Multiple 2 is two fleets fishing with differing selectivity.

Long-term annual yield (tonnes)	Probability of depletion below bound ( $S_{\min} < 0.2\hat{S}_0$ )			Mean spawning stock escapement $\hat{E}$			
	Fleet	Single	Multiple 1	Multiple 2	Single	Multiple 1	Multiple 2
3500		0.015	0.015	0.045	0.807	0.807	0.758
4000		0.026	0.026	0.071	0.779	0.779	0.722
4500		0.046	0.046	0.091	0.751	0.751	0.689
5000		0.065	0.065	0.119	0.726	0.726	0.648
5500		0.082	0.082	0.147	0.695	0.695	0.681
6000		0.107	0.107	0.189	0.664	0.664	0.579

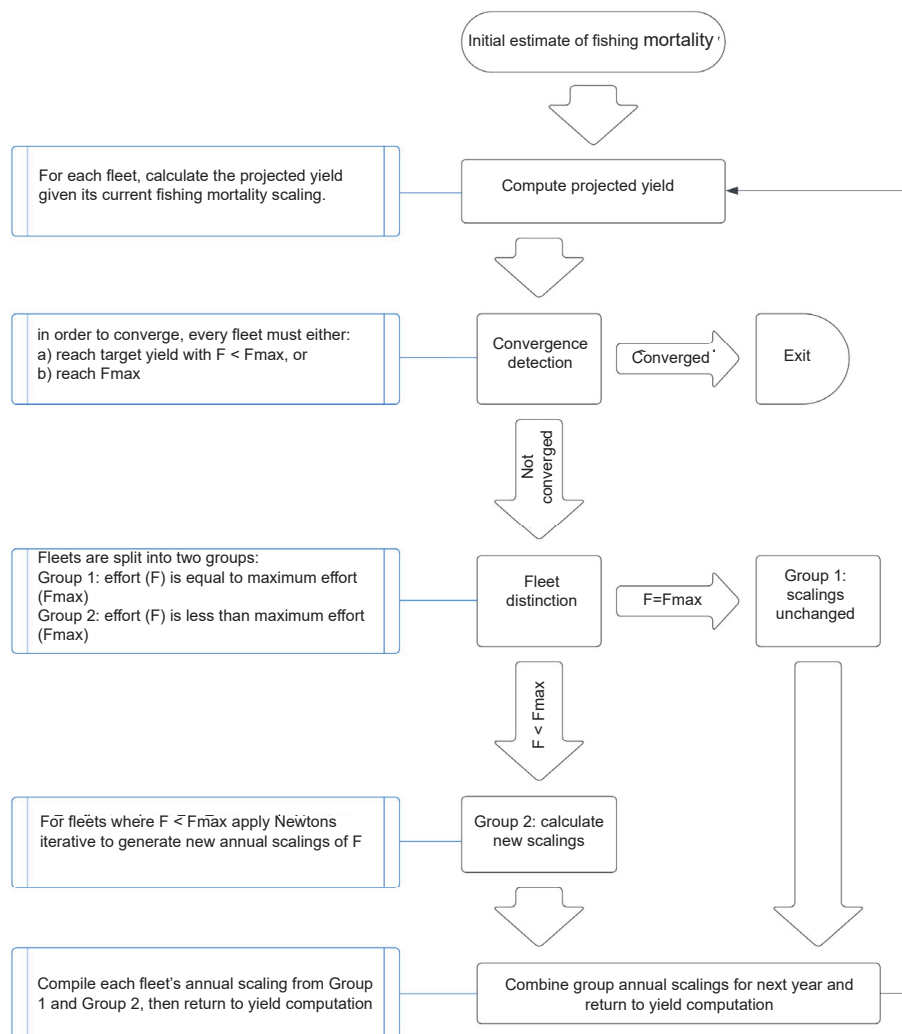


Figure 1: Flow chart of a single determination of the annual scaling of fishing mortality. Blue boxes indicate the processes happening in adjoined steps. The second scenario compared two identical fleets that fish sequentially. The selectivities and target catches are again identical but the fishing season is split in half, with the first fleet fishing only in the first half of the season, and the second fleet fishing only in the second half. In this case, the fleet that fishes first should be able to attain its target catch with less effort. As the target catches increase, the catch for the second fleet should become unattainable before the first, given that the available biomass is depleted by the fishing of the first fleet.

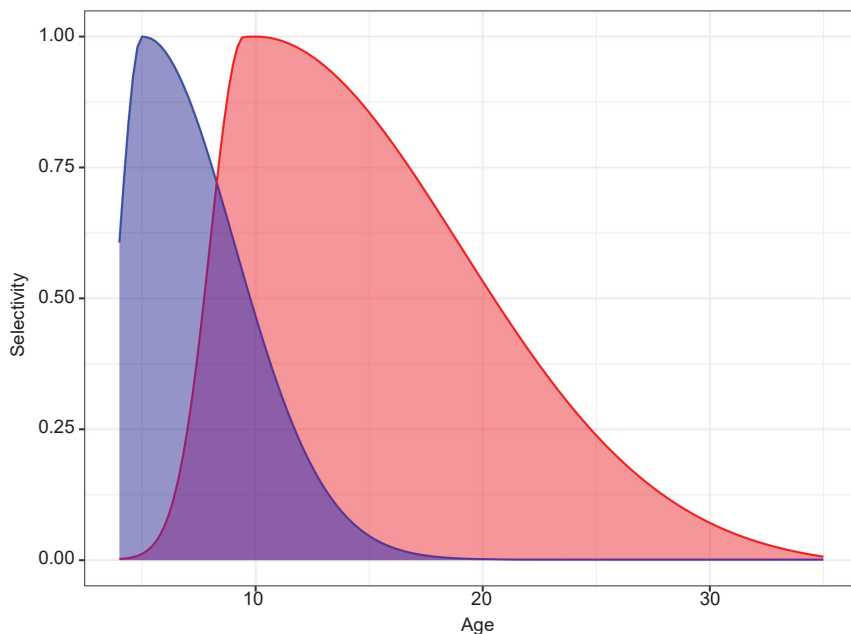


Figure 2: Double normal selectivity used for comparison of trawl (blue) and longline (red) fisheries for *Dissostichus eleginoides*.

#### Liste des tableaux

- Tableau 1 : Fonctionnalités du Grym récemment ajoutées accompagnées d’une brève description.
- Tableau 2 : Paramètres d’entrée des comparaisons d’évaluations de *Dissostichus eleginoides* réalisées dans le Grym.
- Tableau 3 : Comparaison d’exécutions uniques des modèles de légine australe pour des flottilles identiques et séquentielles dans l’exécution de flottilles multiples. La capture est déclarée en grammes pour faciliter la comparaison. L’effort est la mortalité annuelle par pêche, avec un maximum de 4.5.
- Tableau 4 : Comparaison des single modèles d’évaluation de flottilles uniques et multiples avec 10 001 exécutions de projection de six rendements annuels à long terme pour *Dissostichus eleginoides*. « Single » désigne l’ajustement du modèle de flottille unique du Grym, « Multiple 1 » désigne deux flottilles pêchant avec la même sélectivité, et « Multiple 2 » désigne deux flottilles pêchant avec des sélectivités différentes.

#### Liste des figures

- Figure 1 : Organigramme d’une détermination unique de l’échelle de la mortalité par pêche. Les cases bleues indiquent les processus en cours dans les étapes associées. Dans le deuxième scénario, deux flottilles identiques qui pêchent de manière séquentielle sont comparées. Les sélectivités et objectifs de captures sont toujours identiques mais la saison de pêche est divisée en deux, la première flottille pêchant uniquement lors de la première moitié de la saison, et la seconde uniquement lors de la deuxième moitié. Dans ce cas, la flottille pêchant lors de la première moitié de la saison devrait être en mesure d’atteindre l’objectif de capture avec moins d’effort. Lorsque les objectifs de captures augmentent, la capture de la seconde flottille devrait devenir inatteignable avant la première, étant donné que la biomasse disponible est réduite par la pêche menée par la première flottille.
- Figure 2 : Double sélectivité normale utilisée pour comparer la pêche de *Dissostichus eleginoides* menée au chalut (en bleu) et à la palangre (en rouge).

## Список таблиц

- Табл. 1: Новые функции пакета Grym с краткими описаниями.
- Табл. 2: Входные параметры для сравнений оценок Grym, проведенных для *Dissostichus eleginoides*.
- Табл. 3: Сравнение однократных прогонов моделей патагонского клыкача для идентичной и последовательной флотилий на базе версии модели для нескольких флотилий. Вылов указывается в граммах для возможности сравнения. Усилие – это годовая промысловая смертность, не превышающая 4,5.
- Табл. 4: Сравнение моделей оценки с одной и несколькими флотилиями с 10001 прогоном шести долгосрочных годовых уловов *Dissostichus eleginoides*. Single обозначает модель Grym с одной флотилией, Multiple 1 – две флотилии, ведущие промысел с одинаковой селективностью, и Multiple 2 – две флотилии, ведущие промысел с разной селективностью.

## Список рисунков

- Рис. 1: Схема одного определения годового масштаба промысловой смертности. Синими рамками отмечены процессы, происходящие на смежных этапах. Во втором сценарии сравниваются две одинаковые флотилии, которые ведут промысел последовательно. Селективность и целевые выловы снова идентичны, но промысловый сезон разделен пополам, причем первая флотилия ведет промысел только в первой половине сезона, а вторая – только во второй. В этом случае флотилия, ведущая промысел первой, должна быть в состоянии достичь целевого вылова с меньшими усилиями. По мере увеличения целевого вылова вылов для второй флотилии должен стать недостижимым раньше, чем для первой, учитывая, что доступная биомасса истощается в результате промысла первой флотилии.
- Рис. 2: Двойная нормальная селективность, использованная для сравнения тралового (синий) и ярусного (красный) промыслов для *Dissostichus eleginoides*.

## Lista de las tablas

- Tabla 1: Funciones añadidas recientemente al paquete Grym, descritas brevemente.
- Tabla 2: Parámetros de entrada para las evaluaciones de *Dissostichus eleginoides* realizadas con Grym.
- Tabla 3: Comparación de ejecuciones únicas de modelos de austrómerluza negra con flotas idénticas y secuenciales en la implementación de flotas múltiples. La captura se expresa en gramos para facilitar las comparaciones. El esfuerzo es la mortalidad por pesca anual, con un valor máximo de 4,5.
- Tabla 4: Comparación de modelos de evaluación de una y de múltiples flotas con 10001 ejecuciones de proyecciones a futuro de seis rendimientos anuales de *Dissostichus eleginoides* a largo plazo. “Single” indica el ajuste del modelo Grym de flota única, “Multiple 1” indica dos flotas que pescan con la misma selectividad, y “Multiple 2” indica dos flotas con diferentes selectividades.

## Lista de las figuras

- Figura 1: Diagrama de flujo de una determinación única del escalado anual de la mortalidad por pesca. Las cajas azules indican los procedimientos a seguir en cada paso. El segundo escenario compara dos flotas idénticas que pescan secuencialmente. Las selectividades y los objetivos de captura son también idénticos, pero la temporada de pesca se divide en dos, con la primera flota pescando solo en la primera mitad de la temporada y la segunda flota solo en la segunda mitad. En este caso, la flota que pesca primero debería poder alcanzar sus objetivos de captura con un menor esfuerzo. A medida que los objetivos de captura aumentan, la captura de la segunda flota debería ser imposible de alcanzar para esta antes que para la primera flota, dado que la biomasa ya ha sido mermeada por la primera.
- Figura 2: Doble selectividad normal utilizada para la comparación de pesquerías de arrastre (azul) y palangre (rojo) dirigidas a *Dissostichus eleginoides*.

### Grym assessment code for multi-fleets version of project function.

A version of Grym that implements the multiple-fleet functionality is available in the ‘Multi-fleets’ branch of the Github repository: <https://github.com/AustralianAntarcticDivision/Grym>

The new functions are briefly described in this appendix.

In the single-fleet version of Grym, an *annual matrix* is a matrix with one row for the initial time and each subsequent time step through the year, and one column for each modelled age class. In the multiple-fleets version, where appropriate these matrices are generalised to 3-dimensional arrays where again the first and second dimensions correspond to time steps through the year and modelled age classes, and the third dimension corresponds to the separate fleets. The single-fleet implementations of the functions *trapz* and *ctrapz* for computing the composite trapezoidal rule must be generalised to these 3-dimensional inputs. The single-fleet functions *project* and *projectC* must also be extended to compute projections for multiple fleets.

#### *scale3*

The function *scale3* scales a 3D array along its third dimension, optionally summing over the third dimension.

```
scale3 <- function(A,b,sum=FALSE) {
  dm <- dim(A)
  if(sum)
    `dim<-`(`dim<-`(A,c(dm[1]*dm[2],dm[3]))%*%b,dm[1:2])
  else
    `dim<-`(`dim<-`(A,c(dm[1]*dm[2],dm[3]))%*%diag(b),dm)
}
```

#### *trapz*

The *trapz* function from the Grym has been generalised to allow 3D arrays as input.

Given a set of function values  $f(x_k)$  sampled at even increments  $x_k = x_1 + (k-1)h$ , the integral  $\int_{x_1}^{x_n} f(l)dl$  can be approximated by the composite trapezoidal rule:

$$\int_{x_1}^{x_n} f(l)dl = \frac{h}{2}f(x_1) + h \sum_{k=2}^{n-1} f(x_k) + \frac{h}{2}f(x_n)$$

The *trapz* function implements the composite trapezoidal rule. Given a vector *fs* of function values  $f(x_k)$ , *trapz* returns the trapezoidal approximation to  $\int_{x_1}^{x_n} f(l)dl$ . If *fs* is a 2D or 3D array, then each column is assumed to represent a different integrand  $f(x)$ , and *trapz* acts on the columns to return a vector or matrix of approximate integrals.

#### *ctrapz*

The *ctrapz* function from the Grym has been generalised to allow 3D arrays as input. The *ctrapz* function is similar to *trapz* except that it returns the sequence of approximations:

$$\left( \int_{x_1}^{x_2} f(l)dl, \int_{x_1}^{x_2} f(l)dl, \dots, \int_{x_1}^{x_n} f(l)dl \right).$$

In this case, `fs` may be a vector or 2D or 3D array, and it returns a sequence of approximations of the same dimension as the input `fs`.

```
ctrapz <- function(fs,h=1) {
  if(is.array(fs)) {
    dm <- dim(fs)
    Fs <- matrix(fs,dm[1],length(fs)/dm[1])
    for(j in seq_len(ncol(Fs)))
      Fs[-1,j] <- cumsum((h/2)*(Fs[-1,j]+Fs[-nrow(Fs),j]))
    Fs[1,] <- 0
    array(Fs,dm)
  } else {
    c(0,cumsum((h/2)*(fs[-1]+fs[-length(fs)])))
  }
}
```

### *ProjectMF*

The *projectMF* function generalises the *project* function to compute the abundance, biomass and the yields when there are multiple fleets. This function differs from the *project* function in that the inputs relating to the fishing mortalities are specified on a per fleet basis, and a separate yield is computed for each fleet. By assuming the matrix of scaled integrated fishing mortalities, *FFs* has been summed over fleets, and the implementation remains largely unchanged from the single fleet case.

Given inputs

*ws* - matrix of weights at age

*MMs* - matrix of scaled integrated natural mortalities

*FFs* - matrix of summed scaled integrated fishing mortalities, which is the sum of each fleet's matrix of scaled integrated fishing mortalities

*Ffs* - array of scaled fishing mortalities

*Nref* - vector of reference abundances for each age class

*nref* - vector of time steps for which the reference abundance is known

*Bref* - reference biomass

*bref* - vector of time steps for which the reference biomass is known

*yield* - if *yield*=1 only calculate yield for the final time step, if *yield*=2 calculate yield for all time steps, otherwise yield is not calculated at all

The *projectMF* function computes the projections of abundance *N*, biomass *B* and the yield *Y* for each fleet *Y*.

```
projectMF <- function(ws,MMs,FFs=0,Ffs=0,Nref=1,nref=1,Bref=NA,bref=nref,yield=2) {
  if(length(Nref)==1) Nref <- rep.int(Nref,ncol(MMs))
}
```

```

## Integrate N and scale to reference abundance
N <- exp(-MMs-FFs)
for(j in seq_len(ncol(N))) N[,j] <- Nref[j]/mean(N[nref,j])*N[,j]

## Scale by weight at age
B <- ws*N

## Rescale to match reference biomass
if(!is.na(Bref)) {
  r <- Bref/sum(trapzMeans(B[bref,,drop=FALSE]))
  N <- r*N
  B <- r*B
}

## Integrate yield
Y <- switch(yield,
  trapz(Ffs*as.vector(B),1/(nrow(B)-1)),
  ctrapz(Ffs*as.vector(B),1/(nrow(B)-1)))

list(N=N,B=B,Y=Y)
}

```

### *projectCMF*

The *projectCMF* function generalises the *projectC* function to determine annual scalings of fishing effort for each fleet that reproduce known target yields for each fleet, using the modified version of Newton's method described in the text.

#### Given inputs

*ws* - matrix of weights at age

*MMs* - matrix of scaled integrated natural mortalities

*Fs* - array of unscaled integrated fishing mortalities

*fs* - array of unscaled fishing mortalities

*Catch* - target total annual catch

*Nref* - vector of reference abundances for each age class

*nref* - vector of time steps for which the reference abundance is known

*Bref* - reference biomass

*bref* - vector of time steps for which the reference biomass is known

*yield* - if yield=1 only calculate yield for the final time step, if yield=2 calculate yield for all time steps, otherwise yield is not calculated at all

*Fmax* - the maximum F to allow in the search

*tol* - the precision to which to reproduce the observed catch

The *projectCMF* function computes matrices of projected abundance  $N$ , biomass  $B$ , and also projected array of yield  $Y$  and annual scaling of fishing mortalities  $F$  that reproduces the target catch for each fleet.

```
projectCMF <- function(ws,MMs,Fs,fs,Catch,Nref,nref=1,Bref=NA,bref=nref,yield=2,
                      Fmax=4.5,tol=1.0E-6) {
  nfleets <- dim(Fs)[3]
  F <- rep(0,nfleets)
  ## Apply Newton's method

  repeat {
    ## Compute projected yield
    FFs <- scale3(Fs,F,TRUE)
    Ffs <- scale3(fs,F,FALSE)
    pr <- projectMF(ws,MMs,FFs,Ffs,Nref,nref,Bref,bref,yield=1)
    Y <- colSums(pr$Y)

    ## Test for convergence
    if(all(abs(Y- Catch) < tol | (F==Fmax & Y < Catch))) break

    ## Jacobian of the yield
    J <- matrix(0,nfleets,nfleets)
    for(k in 1:nfleets) {
      J[k,k] <- sum(trapz(fs[,k]*as.vector(pr$B),h))
      J[,k] <- J[,k]-colSums(trapz(Ffs*as.vector(pr$B*Fs[,k]),h))
      if(!is.na(Bref))
        J[,k] <- J[,k]+colSums(pr$Y*sum(trapzMeans(Fs[bref,,k,drop=FALSE]*as.vector(pr$B[bref,])))/sum(trap
zMeans(pr$B[bref,,drop=FALSE])))
    }

    ## Protected Newton step
    F1 <- F- solve(J,Y-Catch)
    ok <- (Y >= Catch) | (F1 < Fmax) # | (F < Fmax)
    if(all(ok)) {
      F <- pmin(pmax(F1,0),Fmax)
    } else {
      if(any(ok)) F[ok] <- F[ok]- solve(J[ok,ok,drop=F],(Y-Catch)[ok])
      F[!ok] <- Fmax
    }
  }

  ## Recalculate
  pr <- projectMF(ws,MMs,FFs,Ffs,Nref,nref,Bref,bref,yield)
  pr$F <- F
  pr
}
```